

## Sequences and Series Review

1.  $a_1 = 6; a_n = 4a_{n-1} + 2$

Recursive!  $\hookrightarrow$  term before

$$a_2 = 4(6) + 2 = 26$$

$$\boxed{6, 26, 106, 426, 1706}$$

2.  $a_n = n^2 + 2n$  Explicit!

$$a_1 = (1)^2 + 2(1) = 3$$

$$a_2 = (2)^2 + 2(2) = 8$$

⋮

$$\boxed{3, 8, 15, 24, 35}$$

3.  $\{1 + 2 + 3 + 5 + 8 + 13 \dots\}$

$\hookrightarrow 21$

$$S_7 = 1 + 2 + 3 + 5 + 8 + 13 + 21 = \boxed{53}$$

4.  $\sum_{n=1}^6 n^2 + 3$

$(1)^2 + 3 = 4$	} $4 + 7 + 12 + 19 + 28 + 39$
$(2)^2 + 3 = 7$	
$\vdots$	
$(6)^2 + 3 = 39$	

$$= \boxed{109}$$

5.  $\{-2, -8, -32, -128\}$   $r = 4$

Geometric Sequence

$$a_n = a_1 \cdot r^{n-1}$$

$$\rightarrow \boxed{a_1 = -2}$$
$$\boxed{a_n = (-2) \cdot 4^{n-1}}$$

$$a_{11} = (-2) \cdot 4^{11-1}$$

$$= (-2) \cdot 4^{10} = \boxed{-2,097,152}$$

6.  $\{1, 7, 13, 19, \dots\}$

$$d = 6 \quad a_1 = 1$$

Arithmetic Sequence

$$a_n = 6(n-1) + 1 = \boxed{6n - 5}$$

$$a_n = d(n-1) + a_1$$

$$a_9 = 6(9) - 5 = \boxed{49}$$

4.  $\{81, -27, 9, -3, \dots\}$

$$\frac{-27}{81} = \dots = \frac{1}{3} \rightarrow r = -\frac{1}{3} \quad \text{Geometric Sequence}$$

8.  $\{1, 4, 9, 16, \dots\}$

$[1, 4, 9, \dots]$

9.  $\{20, 16, 12, 8, \dots\}$   $S_{20}$

Arithmetic

$$d = -4$$

$$a_{20} = -4(20-1) + 20 = -56$$

$$S_{20} = 20 \left( \frac{a_1 + a_{20}}{2} \right) = 20 \left( \frac{20 + (-56)}{2} \right)$$

$$\boxed{S_{20} = -360}$$

10.  $\{2, -6, 18, -54, \dots\}$

Geometric

$$a_1 = 2 \quad r = -3 \quad S_8 = a_1 \left( \frac{1 - r^8}{1 - r} \right)$$

$$S_8 = 2 \left( \frac{1 - (-3)^8}{1 - (-3)} \right) = \boxed{-3280}$$

11.

Convergent:  $\{200 + 100 + 50 + 25 + \dots\}$   $r = \frac{1}{2}$

$$\sum_{k=1}^{\infty} 180 \cdot \left( \frac{1}{5} \right)^{k-1} \quad r = \frac{1}{5}$$

$$12. \sum_{k=1}^{\infty} -32 \cdot \left(\frac{1}{4}\right)^{k-1} \quad r = \frac{1}{4} < 1 \therefore \text{convergent}$$

$$a_1 = -32$$

Convergent (geometric)  $\rightarrow S_n = a_1 \left(\frac{1}{1-r}\right)$

$$S_{\infty} = -32 \left(\frac{1}{1-(1/4)}\right) = \boxed{\frac{-128}{3}}$$

$$13. a_1 = -49 \quad a_8 = -14$$

$$a_n = d(n-1) + a_1$$

$$-14 = d(8-1) + (-49)$$

$$-14 = 7d - 49$$

$$35 = 7d$$

$$5 = d$$

↓

$$a_{27} = 5(27-1) + (-49)$$
$$= \boxed{81}$$